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On-line and off-line wheel/rail contact algorithm in the analysis of multibody railroad vehicle systems †

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Abstract

An on-line and off-line hybrid contact algorithm for modeling wheel/rail contact problems is developed based on the elastic contact formulation. In the hybrid algorithm developed in this investigation, the off-line tabular search is used for predicting the location of tread contact points, while the on-line iterative search is used for predicting flange contact points. By so doing, a computationally efficient procedure is achieved while keeping accurate predictions of contact points for severe contact scenarios such as sharp curve and turnout negotiations. The use of the proposed hybrid algorithm can eliminate the time-consuming on-line iterative search for the second points of contact. Since the location of the second point of contact is pre-computed by the contact geometry analysis, the occurrence of two-point contact can be predicted by using the look-up table at the one-point contact configuration. A flange climb simulation demonstrates that the proposed hybrid contact search algorithm can be effectively used for modeling wheel/rail contacts in the analysis of general multibody railroad vehicle systems.

Keywords: Wheel/rail contact; Railroad vehicle dynamics; Multibody dynamics; Contact search

1. Introduction

Predicting the location of contact points is one of the most crucial issues in the analysis of multibody railroad vehicle systems [1, 2]. There are two different approaches used for determining the contact points in the elastic contact formulation. The first approach is called the *on-line* contact search algorithms. The location of contact points is determined online by using iterative procedures at every time step in the dynamics simulation. To this end, contact search is performed by either solving algebraic equations [2] or using nodal search method [3]. On the other hand, in the second approach, *off-line* contact search algorithms, the location of points of contact is pre-computed by the contact geometry analysis and such information is stored in the look-up contact table. The contact point is then predicted by interpolation of the table data for given wheelset displacements obtained in the dynamic simulation. Such an off-line search method has been widely used in the specialized railroad vehicle dynamics codes, while the on-line search is used in general multibody computer algorithms. Although time-consuming on-line iterative procedures can be avoided in the off-line approach, special care needs to be exercised since rigid contacts are usually assumed in the off-line contact geometry analysis. This leads to inconsistent prediction of contact points when severe contact scenarios such as multiple flange contacts in sharp curve and turnout negotiations are considered. It is, therefore, the objective of this investigation to develop a nu-

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merical procedure that can be used for predicting the tread and flange contacts in an efficient way. To this end, the on-line and off-line hybrid contact search algorithm is developed.

2. Wheel and rail coordinate systems

2.1 Wheelset coordinate system

The global position vector of an arbitrary point of contact on wheelset *w* can be defined as [2]

$$\mathbf{r}^{wk} = \mathbf{R}^{w} + \mathbf{A}^{wi}\bar{\mathbf{u}}^{wk} \tag{1}$$

where superscript *k* denotes the contact number (*k*=1 for the right wheel, *k*=2 for the left wheel, etc); the position vector $\mathbf{R}^{w}=[R_X^w \quad R_Y^w \quad R_Z^w]^T$ defines the global position of the origin of the wheelset coordinate system; $\mathbf{A}^{wi}$ defines the coordinate system of the wheelset that does not rotate with the wheelset about Y^w-axis. Such a coordinate system is called *wheelset intermediate coordinate system*, while the orientation of the wheelset with rotation about Y^w-axis can be defined by using the three Euler angles $\boldsymbol{\theta}^w=[\psi^w \quad \phi^w \quad \theta^w]^T$ (yaw angle ψ^w, roll angle ϕ^w and pitch angle θ^w); and $\bar{\mathbf{u}}^{wk}$ is the position vector that defines the location of contact point defined with respect to the wheelset intermediate coordinate system. The profile coordinate system $X^{wk}Y^{wk}Z^{wk}$ (*k*=1,2) is defined in order to parameterize the wheel geometry as shown in Fig. 1. Using the wheel profile coordinate system, the lateral and circumferential surface parameters given by s_1^{wk} and s_2^{wk} are introduced as shown in Fig. 1. With

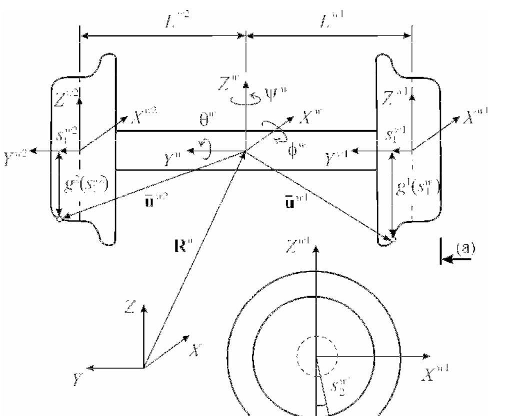

Fig. 1. Parameterization of wheelset.

these two parameters, the rolling radius of each wheel is given by $g^k(s_1^{wk}, s_2^{wk})$. If the rolling radius can be assumed to be constant in the circumferential direction of the wheel, one can have $g^k = g^k(s_1^{wk})$. Furthermore, the two tangents at the contact point along the lateral (s_1^{wk}) and circumferential (s_2^{wk}) directions as well as the normal are given as

$$\bar{\mathbf{t}}_1^{wk} = \frac{\partial \bar{\mathbf{u}}^{wk}}{\partial s_1^{wk}}, \quad \bar{\mathbf{t}}_2^{wk} = \frac{\partial \bar{\mathbf{u}}^{wk}}{\partial s_2^{wk}}, \quad \bar{\mathbf{n}}^{wk} = \bar{\mathbf{t}}_1^{wk} \times \bar{\mathbf{t}}_2^{wk} \tag{2}$$

Differentiating Eq. (1) with respect to time, the absolute velocity vector of the arbitrary point of contact on the wheelset can be written as

$$\dot{\mathbf{r}}^{wk} = \dot{\mathbf{R}}^{w} + \boldsymbol{\omega}^{w} \times \mathbf{u}^{wk} \tag{3}$$

where $\boldsymbol{\omega}^{w}$ is the absolute angular velocity vector of the wheelset and $\mathbf{u}^{wk}$ is the absolute position vector of the arbitrary point of contact.

2.2 Rail coordinate system

As shown in Fig. 2, the profile coordinate system of rail *r* is defined for right and left rails as $X^{rk}Y^{rk}Z^{rk}$. Using this profile coordinate system, the longitudinal and lateral surface parameters given by s_1^{rk} and s_2^{rk} are introduced as shown in Fig. 2. With these two parameters, the cross-sectional shape of the rail can be defined by $f^k(s_1^{rk}, s_2^{rk})$. If the shape of the rail can be assumed to remain constant along the longitudinal axis of the rail, one can have $f^k = f^k(s_2^{rk})$. Assuming that the rail is rigidly fixed to the global coordinate system, the global position vector of the contact point on the rail can be defined as [2]

$$\mathbf{r}^{rk} = \mathbf{R}^{rk} + \mathbf{A}^{rk}\bar{\mathbf{u}}^{rk} \tag{4}$$

where $\mathbf{R}^{rk}$ defines the location of the origin of the rail profile coordinate system defined with respect to the global coordinate system and is given as a function of the longitudinal surface parameter s_1^{rk}; $\mathbf{A}^{rk}$ defines the orientation of the profile coordinate system that is also a function of the longitudinal surface parameter s_1^{rk}; and $\bar{\mathbf{u}}^{rk}$ is the location of the contact point defined with respect to the profile coordinate system. In a way similar to Eq. (2), the tangents along the longitudinal and lateral directions as well as the normal can be obtained.

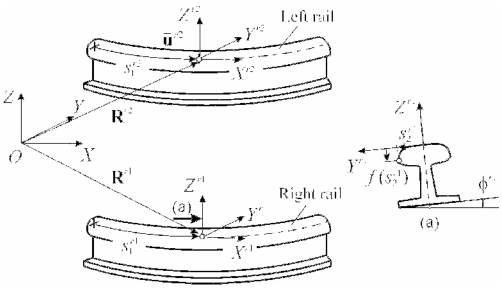

Fig. 2. Parameterization of rail.

3. Contact search methods

3.1 On-line search

In the on-line contact formulation, the location of contact points is determined online by using iterative procedures. Since the location of the contact points is predicted online, contact points can be accurately and reliably predicted in this algorithm. This is one of the reasons why general multibody computer algorithms employ on-line based algorithms. In the elastic contact formulation, two different approaches are used for determining the location of contact points. In the first approach, wheel and rail profiles are parameterized by discrete nodal points, and the distance between nodes on wheel and rail profiles is used to determine the contact point. This method has an advantage in that requirements of surface smoothness are not necessary since derivatives of wheel and rail profile functions are not used for predicting contact points.

In the second approach, the following four algebraic equations are solved to determine the four surface parameters [2]:

$$\mathbf{E}^k(\mathbf{s}^{wrk}) = \begin{bmatrix} \mathbf{t}_1^r \cdot (\mathbf{r}_P^w - \mathbf{r}_P^r) \\ \mathbf{t}_2^r \cdot (\mathbf{r}_P^w - \mathbf{r}_P^r) \\ \mathbf{t}_1^w \cdot \mathbf{n}^r \\ \mathbf{t}_2^w \cdot \mathbf{n}^r \end{bmatrix}^k = \mathbf{0} \tag{5}$$

where $\mathbf{s}^{wrk}$ is a set of surface parameters for contact k given by $\mathbf{s}^{wrk} = [s_1^{wk} \quad s_2^{wk} \quad s_1^{rk} \quad s_2^{rk}]^T$. As shown in Fig. 3(a), the contact point that satisfies Eq. (5) guarantees that tangent planes defined at the points on the wheel and rail are parallel (tangency condition); and the relative distance projected on the tangent plane of the rail is always zero. The location of the contact point calculated by Eq. (5) is generally

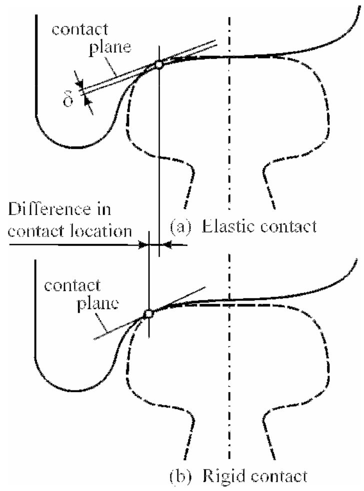

Fig. 3. Elastic and rigid contacts.

more accurate than those obtained by using the nodal search method, since discrete nodal points are not used for predicting the location of contact points. Having obtained the contact point, the local deformation (indentation) at the contact point is given as

$$\delta^k = \mathbf{n}^{rk} \cdot (\mathbf{r}_P^{wk} - \mathbf{r}_P^{rk}) \tag{6}$$

and the normal contact force can be defined by the Hertz's contact model as [3]

$$F_n^k = -K(\delta^k)^{3/2} - D\dot{\delta}^k \tag{7}$$

where K is Hertz's constant that depends on the surface curvature and material properties, and D is the damping coefficient. Having obtained the normal contact force, the tangential creep forces need to be calculated. In this investigation, FASTSIM algorithm developed by Kalker is used to calculate the creep forces on the tread and flange of wheels [4].

3.2 Off-line search

Despite the accurate prediction of the location of contact points, the use of the on-line contact search algorithm is computationally expensive due to the use of iterative procedures. For this reason, the off-line contact algorithm is conventionally used in the specialized railroad vehicle dynamics codes in order to reduce the computational effort for contact search [1]. In this approach, the location of contact points is precomputed by the contact geometry analysis and a

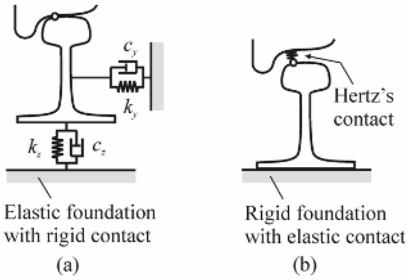

Fig. 4. Schematic representation of the (a) flexible track foundation model and (b) Hertz's contact model.

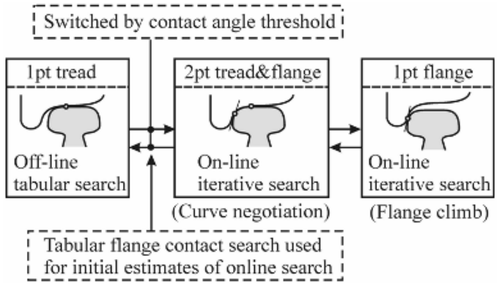

Fig. 5. On-line/off-line hybrid algorithm.

look-up contact table is generated, from which the location of contact points is computed in the dynamic simulation. To generate the look-up contact table, the non-conformal contact condition that guarantees the point and tangency conditions between two bodies in contact is imposed as follows:

$$\mathbf{C}^k(\mathbf{q}^w, \mathbf{s}^{wrk}) = \begin{bmatrix} \mathbf{r}_P^w - \mathbf{r}_P^r \\ \mathbf{t}_1^w \cdot \mathbf{n}^r \\ \mathbf{t}_2^w \cdot \mathbf{n}^r \end{bmatrix}^k = \mathbf{0} \tag{8}$$

The configuration of two bodies that satisfies the preceding equations is schematically shown in Fig. 3 (b). Since a non-conformal contact condition is imposed on each contact point, *rigid* contact is automatically assumed [5]. For this reason, in the use of elastic contact formulations with look-up contact tables, the normal contact forces are generally calculated by using a track foundation model as shown in Fig. 4(a) [1]. In such a case, Hertz's contact model shown in Fig. 4(b) is not used between the wheel and rail, which sometimes leads to inaccurate prediction of contact forces when severe contact conditions such as flange contact/impact are encountered [6]. For this reason, in this investigation, Hertz's contact model is used for computing the normal contact force and the track foundation model is not used to determine the normal contact force.

It should be noted that the contact point predicted by the look-up table is based on the assumption of rigid contacts and the indentation between the two body surfaces is not allowed, while in the elastic contact formulation, small deformations between the two bodies are allowed to determine the normal contact force with Hertz's theory. For this reason, it is not guaranteed that contact points predicted by the look-up table coincide with those obtained by solving Eq. (5) in the on-line iterative search. In other words, if a vector of the relative distance between the points of contact is approximately perpendicular to the tangent contact plane, the condition of Eq. (5) can be satisfied. In the case of tread contacts, such a condition can be automatically satisfied since the contact angle is generally very small, while in the case of flange contact that has large contact angle, the location of contact point given under the assumption of rigid and elastic contacts can be different. Such configuration is schematically demonstrated in Fig. 3. Special care, therefore, needs to be exercised for predicting consistent points of contact when the off-line contact search is used with Hertz's contact model.

3.3 Hybrid search

The tabular off-line contact search can be effectively used for treating tread contacts, while the off-line search is no longer valid since the contact indentation contributes to the wheelset lateral and yaw displacements that are used as inputs of the look-up table (see Fig. 3). To overcome this problem, the off-line search that can be used for tread contact problems is switched to the on-line search when the contact point jumps to the flange region as shown in Fig. 5. With the hybrid use of the on-line and off-line contact search, a computationally efficient algorithm can be achieved while keeping the accurate prediction of contact points on the tread and flange.

Furthermore, the use of the hybrid algorithm eliminates the on-line detection of the second point of contact as well. The contact detection of the second point of contact is a time-consuming part of existing contact formulations. Since the location of the second point of contact is pre-computed by the contact geometry analysis, the occurrence of the second point of contact can be predicted by the look-up table in a straightforward manner. The point predicted by the

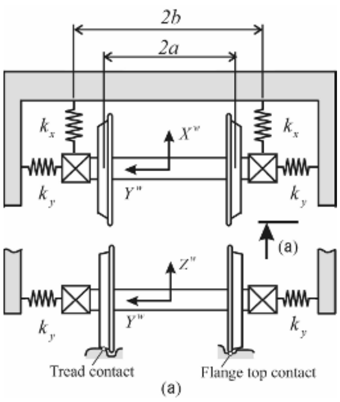

Fig. 6. Suspended wheelset model.

table is then used as an initial estimate for the on-line iterative procedure to achieve accurate predictions. This leads to faster convergence in the iterative procedure as well. For the two-point contact scenarios encountered in curve negotiations, the on-line search is used for both flange and tread contacts. The on-line one-point contact is also important for flange climb scenarios, as will be discussed in the numerical example.

4. Numerical examples

In this section, a flange climb scenario in turnout sections is considered in order to demonstrate the use of the on-line and off-line hybrid contact search algorithms. The dimension of the suspended wheelset used in this example is shown in Fig. 6, and the wheelset mass and the moments of inertia are assumed to be $m^w = 1568$ kg, $\bar{I}^w_{xx} = \bar{I}^w_{zz}$=656 kgm^2, and $\bar{I}^w_{yy} = 168$ kgm^2. A downward vertical load of 50 kN is applied to the wheelset center. The spring constants and damping coefficients of the primary suspensions are assumed to be $k_x = 1\times10^5$ N/m, $k_y = 2\times10^5$ N/m, $c_x = c_y = 0$ Ns/m, while $2a = 1.105$ m, $2b = 1.305$ m are used in Fig. 6. The track gauge is 1.067 m and the wheel and rail are profiled by a cubic spline function.

In this simulation scenario, the right wheel is in contact with the rail at the top of the flange, while the left wheel has tread contact at the initial configuration. Due to the large difference in rolling radii, the wheelset is steered in the direction of the wheel with smaller rolling radius and a flange contact occurs on the left wheel. The lateral and vertical displacements of the wheelset are shown in Fig. 7. It can be seen from this Fig. that the wheelset moves left (positive) due to the rolling radius difference and then the wheel

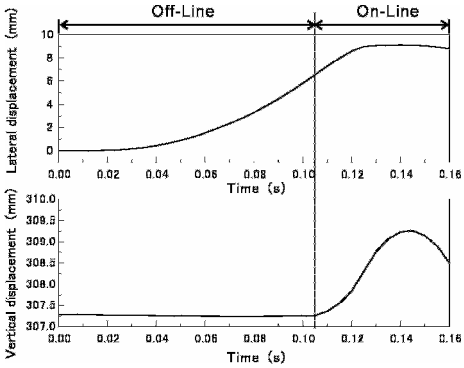

Fig. 7. Wheelset lateral and vertical displacements.

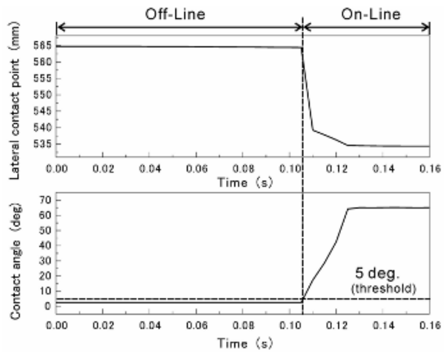

Fig. 8. Contact point and the contact angle (Left wheel).

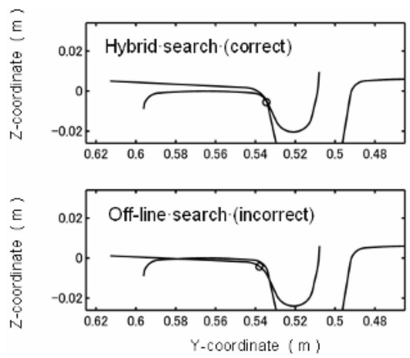

Fig. 9. Configuration of wheel/rail.

starts climbing the rail after around 0.105 s. This is when the contact point jumps from the tread to flange. After the flange contact occurs, the vertical displacement of the wheelset increases and reaches the maximum point around 0.14 s. In order to see the change in the location of the contact point, the global position of the contact point in the lateral direction and the contact angle of the left wheel are shown in Fig. 8. It can be observed from these figures that the jump in the location of contact point occurs around 0.105 s and the contact angle increases up to the flange angle (65 degree) as the wheel climbs. Since the threshold of the contact angle used for switching the off-line and on-line searches is assumed to be 5.0 degrees, the contact search around the flange region is performed by the on-line approach after around 0.105s.

Fig. 9 compares the wheel and rail configurations at 0.13 s obtained by using the hybrid and the off-line algorithms. As discussed in the previous section, the incorrect contact point is predicted when the off-line approach is solely used for modeling the flange contact. As a result, the wheel and rail are erroneously interpenetrating without the flange climb. Recall that, for the flange contact, a small change in the lateral displacement leads to a significant change in the location of contact point due to the large contact angle. This problem can be circumvented by using the switching algorithm proposed in this investigation and the flange climb is correctly simulated.

5. Summary and conclusions

In this investigation, an on-line and off-line hybrid contact algorithm for modeling wheel/rail contacts is developed for the analysis of multibody railroad vehicle systems. It is shown that a tabular off-line contact search can be effectively used for treating tread contacts, while this method with Hertz's contact model is no longer valid when the flange contact occurs since the contact indentation contributes to the wheelset lateral and yaw displacements that are used as inputs of the look-up table. In the proposed approach, the off-line search used for the tread contact is switched to the on-line search when the contact point jumps to the flange region and the contact point predicted by the table is used as an initial estimate for the on-line iterative procedure to improve the numerical convergence. Furthermore, the use of the hybrid algorithm eliminates the time-consuming on-line detection of the second point of contact. Numerical results are presented in order to demonstrate the use of the contact algorithm developed in this investigation.

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